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AN INVESTIGATION OF STUDENTS' QUANTITATIVE REASONING THROUGH MODELING PROCESS

Research article

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Abstract

Quantitative reasoning is defined as reasoning about relationships between items, measurements of objects, and quantities rather than numbers. Both in the transition from arithmetic to algebra and in the problem-solving process, quantitative reasoning is seen as a critical instrument for the development of students' mathematical skills. In the development of quantitative thinking, problems based on daily life experiences play a significant role. The mathematical modeling process is critical in generating mathematical solutions to challenges encountered in everyday life. The goal of this study is to look at how students' quantitative reasoning structures change over time as a result of modeling. A case study was used to assess the research's reasoning process. The study included students with poor, moderate, and strong mathematical reasoning abilities. Over the course of five weeks, the participants were exposed to the modeling process as a group once a week. Clinical interviews with the students were done before and after the modeling process in the study. The modeling process was recorded, observation notes were gathered, and the students' solution sheets were collected. The study's data analysis were carried out in two stages: during data collection and after data collection. The findings reveal significant changes in students' quantitative reasoning structures. The study makes crucial recommendations for both theoretical and practical research.

Keywords: quantitative reasoning, mathematical modeling, problem solving, mathematics education

1. Introduction

With the developing technology in today's world, individuals are expected to be good problem solvers. Individuals must be able to identify variables in a situation, understand relationships between variables, analyze and interpret the situation in order to make logical and correct decisions in daily , academic and business life, and to solve problems, implying that they have reached a certain level of quantitative reasoning (Agustin, Agustin, Brunkow & Thomas , 2012). A rising corpus of research (e.g., Ellis 2007; Moore 2012, 2014; Moore and Carlson 2012; Tallman & Frank, 2020) has recognized quantitative reasoning as a powerful mode of thinking that is critical to developing productive understandings of mathematical concepts. Quantitative reasoning is the ability to connect, reason about, and solve problems involving quantities, which are features of situations that can be quantified (Moore, Carlson & Oehrtman, 2009; Thompson 1988). Quantitative reasoning is defined as reasoning about relationships between items, measurements of objects, and quantities rather than numbers (Thompson, 1995). Making sense of relationships between quantities is crucial in quantitative reasoning (Mkhatshwa, 2020; Smith & Thompson, 2007; Thompson, 1993). The student can use quantitative reasoning to perceive a situation, construct quantities linked to the circumstance, build a relationship between the numbers established, and analyze the situation (Moore et al., 2009). Individuals' conceptual development is aided by representing general links between quantitative reasoning and quantities. When a person engages in quantitative reasoning, he gains a better knowledge of how to apply numbers and the mathematization process to real-world situations and generates/uses logical arguments in the process (Powell & Leveson, 2004). The mental processes used in quantitative reasoning support the development of students' reasoning regarding their mathematical understanding and enable them to reflect on them (Moore et al., 2009). Quantitative reasoning helps students make sense of a problem related to mathematics, create the quantities in this context, associate these quantities with each other, manipulate them and use them in accordance with the problem situation, and describe their mental actions in this process (Weber, Ellis, Kulow and Ozgur, 2014). Quantitative structures arising from mental actions support students in developing meaningful formulas, calculations and graphs (Moore, 2010).

Problems based on daily life experiences have an important place in the development of quantitative reasoning. Smith and Thompson (2007) emphasize that in order to develop quantitative reasoning skills, the problems involving real-life situations in which students judge and conceptualize inter-quantitative mathematics levels should be addressed at the level of mathematics. The findings of the literature study also demonstrate that by developing quantitative reasoning skills, students' cognitive challenges in the process of addressing complicated issues can be decreased, and concepts can be better understood (Ellis, 2007; Moore, 2010; Smith & Thompson, 2007; Thompson, 1988; Thompson, 1993). In a study conducted by Moore and Carlson (2012) on university students, it was observed that students who developed quantitative reasoning skills in the process of solving daily life problems were able to solve problems more meaningfully. The mathematical modeling process plays an important role in producing solutions from a mathematical perspective to the problems encountered in daily life.

1.1. Theoretical Framework

The mathematical modeling process can be generally defined as the cycle that includes the sub-processes of producing mathematical solutions by transforming a daily life problem into a mathematical problem and interpreting these mathematical solutions in the context of daily life (Berry & Houstan, 1995; Blum & LeiB, 2007; Borromeo-Ferri, 2006; Lesh & Doerr, 2003). Since every mathematical model is a product of the mathematical modeling process, one of the most important topics in mathematics learning and teaching literature is the mathematical modeling process (Kaiser, Blomhøj & Sriraman, 2006). It was revealed that mathematical modeling cycles (Blum & Leiß, 2007; Kaiser & Sriraman, 2006) and mathematical modeling competencies (Blum, 2002; Maaß, 2006; Niss, Blum & Galbraith, 2007) are important in revealing the mathematical modeling process. The reason is that mathematical modeling cycles and competencies are the components that provide an understanding of ideal behaviors in the mathematical modeling process (Haines & Crouch, 2010). When looking at studies on mathematical modeling, it's clear that it's viewed from two separate perspectives in instructional programs and applications. One perspective regards mathematical modeling as a tool for demonstrating, developing, and motivating pupils in connection to specific mathematical settings (Chinnappan, 2010). Another viewpoint sees mathematical modeling as a means of achieving educational objectives rather than a tool for the creation of some mathematical understanding at the end of a lesson. Both perspectives acknowledge that mathematical modeling is a general process involving components such as formulation, mathematization, solution, interpretation, and evaluation (Stillman, 2012). Mathematical modeling cycles are expressed as a cyclical process in which the real state (modeling state), mental representation of the state (state model), real model, mathematical model, mathematical result and real result stages (Borromeo Ferri, 2006).

Both in the transition from arithmetic to algebra and in the problem-solving process, quantitative reasoning is seen as a critical instrument for the development of students' mathematical skills (Weber et al., 2014; Moore, 2010). The contribution of quantitative reasoning to thinking styles such as arithmetical, algebraic, and functional thinking should be learned by children from an early age, according to the research (Smith & Thompson, 2007; Thompson, 1988). Research shows the importance of gaining quantitative reasoning skills especially for students at elementary school level (Ellis, 2007; Smith & Thompson, 2007). The majority of studies with elementary school students in the algebra process have shown that students who lack quantitative reasoning skills struggle to interpret problems that require reasoning, such as daily life problems, and instead resort to arithmetic rather than establishing inter-quantitative relationships. However, students who possess quantitative reasoning skills do not require symbolic representation (Kabael & Akın, 2016; Smith & Thompson, 2007). Given the importance of problems based on daily life experiences in the formation of quantitative reasoning, the goal of this study is to look at how students' quantitative reasoning structures evolve through the modeling process. The study's research questions are listed below.

1. How is the student's (who is regarded to have poor quantitative reasoning) quantitative reasoning structure evolve through modeling process?
2. How is the student's (who is regarded to have moderate quantitative reasoning) quantitative reasoning structure evolve through modeling process?
3. How is the student's (who is regarded to have strong quantitative reasoning) quantitative reasoning structure evolve through modeling process?
4. How is the group's quantitative reasoning structure evolve in modeling process?

2. Method

In this study, it will be described how to evaluate students' quantitative reasoning structures in depth and detail rather than making broad generalizations about their intellectual processes, and how to explain the network that influences these processes using a methodical way. Therefore, due to the nature of this research, a large amount of data is obtained in the research with a small number of participating students. For this reason, the reasoning process in the research has been examined with case study. This research method is used to define and see the details that make up a situation, to develop possible explanations about a situation and to evaluate a situation (Gall, Gall & Borg, 2007; Yin, 2003)

2.1.Participants

Participants of the study are three students. The students were selected from students who attend eighth grade at an elementary school located in a city center in Turkey. Criterion sampling, which is one of the purposeful sampling methods, was used (Yıldırım & Şimşek, 2011) and the quantitative reasoning skills of the students were considered as criteria. In this context, an average of 30-40 minutes of clinical interviews were conducted with 18 eighth grade students. Based on these interviews, students who are regarded to have poor, moderate and strong quantitative reasoning were identified. Considering their volunteering to participate in the study, a female student who is regarded to have poor quantitative reasoning, a male student who is regarded to have moderate quantitative reasoning, and a female student who is regarded to have strong quantitative reasoning were identified as participants of the study. The reason why students with different quantitative reasoning are chosen is to examine the change of quantitative reasoning in the application process for each level. Since students in the classes will have varying levels of quantitative reasoning skills, it is critical to assess them at all levels.

2.2. Data Collection

The participants experienced modeling process as a group once a week over a five-week period. The participants produced solution approaches in a modeling activity every week as a group. 5 modeling activities implemented were selected from the book “Modeling Questions from Daily Life” (Erbaş, Çetinkaya, Çakıroğlu, Aydoğan Yenmez, Şen, Korkmaz, Kertil, Didiş, Baş, & Şahin, 2016), which includes activities developed by a project group, and pilot activities and practices were realized. The activities were determined by two mathematics education experts among the 45 activities included in the book, taking into account the criteria to actively use quantitative reasoning. In these modeling processes, there was a researcher and an expert observer in mathematics education. During the modeling process, no guidance was given on the solution, and guidance was given to the participants on what to expect and what their task was at the event. In this context, the participants were asked to examine questions (such as "Why did you think so?" "How did you come to this conclusion?") to encourage participants to express how they think about the modeling problem, what kind of ideas they develop, how they approach the problem. In addition, the quantitative reasonings of the students were provided to listen carefully without evaluating them, and when they entered the vicious circle in the thinking process, they were asked to look from different directions. During the solution process, observation notes were kept and interviews were held with group members. At the end of each modeling process, which lasted about 60-70 minutes, the group was asked to present their solutions. The whole process was recorded on video and worksheets used by the group in the solution process were collected.

In the study, clinical interviews were conducted with the students before and after the modeling process, including quantitative difference, complex additive situations, combination of differences and real life problems in the type of quantitative ratio that may reveal their quantitative reasoning skills. In this process, the problems in the Tanışlı and Dur (2018) studies were used. In problem situations involving quantitative difference, it is possible to obtain a new quantity as a result of comparing two quantities collectively, and in problem cases containing quantitative difference, the two quantities are multiplied by multiplication. While problem states involving complex additive states involve one or more quantitative differences, there are at least six quantitative and three quantitative operations in problem situations with relational complex states (Smith & Thompson, 2007; Tanıslı & Dur, 2018). Different questions with similar content were prepared for the pre and post-clinical interviews. The prepared clinical interview questions were presented to the opinions of three expert mathematics educators. Then, in order to test the comprehensibility of the questions and to develop the questions, a pilot study was carried out with three eighth grade students. The students were determined according to their success levels by taking into account the mathematics grade report scores and the opinions of the teachers. A pilot study was conducted on three volunteer students with strong, moderate and poor success levels. Clinical interviews with each student were completed in 3 days, not exceeding an average of 30-40 minutes. Necessary arrangements were made by correcting expressions that were not understood on questions.

2.3. Data Analysis

The study's data analysis were carried out in two stages: during data collection and after data collection. All analysis from the first day of the students' meeting until the last day of data collection was included in the data analysis. Following data collection, all analyses of data collected from students were grouped. Furthermore, data analysis was split into two categories: within-case and cross-case analysis (Merriam, 2002; Miles & Huberman, 1994). Since the study's focus was both within and across instances, with-in case and cross-case analysis methodologies were appropriate with the study's focus (Gerring, 2007). With-in cases analyses were conducted on the data set for each student. The cross-case analyses were built on the results from with-in case analyses. Analysis for each case was also supported by the comparison analysis that used to design for generalizations across students (Merriam, 2002; Strauss & Corbin, 1998). Comparisons for the cross-case analyses began during the data collection since the researcher spent time with all students during that period. In addition, comparisons took place as the researcher received larger sets of data and gain more experience within settings. In with-in case analyses, changing quantitative modeling structures of the students were examined by focusing on the data obtained from the pre and post clinical interviews of each students. In cross-case analyses, the focus is on the quantitative reasoning structures that are introduced and changed in each modeling process. In the analysis of quantitative reasoning structures, themes and contents were created primarily by using the literature (Carlson & Oehrtman, 2005; Harel, 2013; Moore, 2011; Moore, Carlson & Oehrtman, 2009; Thompson, 1988; Weber et al., 2014). These themes are: (i)Determining and interpreting quantities in the context of the problem,(ii) determining the quantitative relations in the context of the problem and analyzing the change of the quantities,(iii) determining and calculating the quantitative processes in the context of the problem, (iv) interpreting the quantitative information in the context of the problem, (v) checking and drawing conclusions. Each transcripts and field notes were analyzed in order to get the descriptive codes for summarizing the segments of data and to provide the bases for later higher order coding which is pattern (inferential) codes (Miles & Huberman, 1994). After the descriptive codes, the pattern coding began in order to put together the descriptive codes to more meaningful units. Then these meaningful units, which were relating with each other, were organized together and the themes that are determined based on the literature were decided.

To achieve reliability of the data analysis, data were coded by the researcher and a mathematics educator specialized in quantitative reasoning independently. Number of agreements and disagreements were identified for the code, the formulation of Miles and Huberman (1994) “Reliability = Agreement / (Agreement + Disagreement)” was used, and reliability values of the data analyses were found 89%. The reliability calculations above 70% are accepted as reliable for the research, this result is accepted as reliable for the related research.

3. Findings

According to pre and post clinical interviews, the change in students' quantitative reasoning structures is examined under the headings below according to the students who are regarded to have poor , moderate and strong quantitative reasoning. The student who is regarded to have poor quantitative reasoning was identified as QR1, the student who is regarded to have moderate quantitative reasoning was identified as QR2, the student who is regarded to have strong quantitative reasoning was identified as QR3, and the researcher was identified as R.

3.1. The Student who is Regarded to have Poor Quantitative Reasoning

The following findings were reached under the theme of determining and interpreting the quantities in the problem context of the student who is regarded to have poor quantitative reasoning.

In the pre-clinical interview, it was determined that the student had difficulty in determining the quantities related to the problem, could not grasp the quantifiable properties and unit of quantities, could not distinguish the quantities and the unit of quantities, and did not interpret the quantities related to the solution of the problem in a meaningful way (expressing quantities by considering the object, the object's measurable property and its unit together). In the clinical interview after the application, it was determined that it was able to determine the quantities, distinguish the units of quantities and interpret the relevant quantities in a meaningful way. It can be said that there is a positive development in the quantitative reasoning structure of the student under this theme.

The following findings were reached under the theme of identifying quantitative relationships in the context of the problem and analyzing the change of quantities.

It was found that the student had difficulty in making relational, multiplicative and proportional reasonings about relationships between quantities and having difficulty in establishing relationships between quantities in the context of the problem in order to find the unknown value and estimate a series of calculations before calculating in the pre-clinical interview. It has been found that a model, formula and algebraic sentence cannot express and represent all the quantities and relationships between quantities. It was determined that she could not analyze the change of the quantities in the context of the problem and could not interpret the value of one variable together with the change in the other variable. In the clinical interview conducted after the application, it was found that relational reasoning between quantitative relations and cumulative relations could be made, but difficulties were still encountered in complex situations involving multiplicative and proportional reasoning, and quantitative relations in the context of the problem could be established. It has been found that all the quantities and their relationships between quantities can be expressed and represented, except for problems involving a model, formula, and complex case in algebraic sentence. It has been determined that the change of quantities in the context of problems can be analyzed, except for problems involving complex situations, and the value of one variable can be interpreted with the change in the other variable.

The following findings were reached under the theme of identifying and calculating quantitative operations in the context of the problem.

It was determined that the student could not use the quantities in accordance with the problem context in the pre-clinical interview and had difficulty in handling and manipulating the quantities. It was revealed that she did not know how to calculate the quantities in the context of the problem and could not describe the situation. It was observed that she could not explain the calculations made in terms of the quantities and the relations between the quantities in the context of the problem. It has been determined that it cannot determine the meaning of the calculated value and how it is related to the purpose of the problem. In the clinical interview conducted after the application, except for the problems involving complex situations, quantities were used in accordance with the problem context, it was able to carry out operations on quantities, how to calculate the quantities in the context of the problem, and the calculations made in terms of the quantities and relationships between the quantities in the context of the problem. It has been observed that it can be explained and the meaning of the calculated value and how this value relates to the purpose of the problem.

The following findings were reached under the theme of interpreting, controlling and inferring quantitative information with the context of the problem.

In the pre-clinical interview, it was observed that the student could not review and retest every aspect of the solution plan in terms of quantitative relationship. It has been determined that the calculations made in terms of the relationship between quantities and quantitative coordination are not controlled, and when it finds an unexpected value, the situation in the context of the problem is not returned. It has been found that by reference to the quantities and the problem context in the context of the problem, the rationale for the correctness of the solution cannot be presented or an argument cannot be created. In the clinical interview after the application, except for the problems involving complex situations, in terms of quantitative relation, every aspect of the solution plan can be revised and retested, the calculations made between quantitative relations and quantitative coordination are checked, and when it finds an unexpected value, it is returned to the situation in the context of the problem. It was found that the reason for the correctness of the solution was presented or an argument was created by referring the quantities and problem context in the context of the solution.

In the pre-clinical interview, she could not determine the relations between the quantities required for the problem in the real life problem, which included a quantitative difference, did not distinguish the quantity unit and interpreted the quantities in a meaningful way. It was determined that she could not use quantities in accordance with the problem context, had difficulty in handling and manipulating quantities. The student's answer (Fig. 1) and the dialogue section from the clinical interview were given below.

Real life problem with quantitative difference

Eda, Zeynep and Miray are three friends and always have their meals together at school. If the number of meals loaded on one of the cards decreases, the other two friends load additional meals to that person. The number of meals taken by each other within a certain period of time is as follows:

(Note: Eda, Zeynep and Miray do not load meals on their cards from other friends during this period and do not eat anywhere else.)

1) Eda received 14 from Zeynep, 20 from Miray; Zeynep received 26 from Eda, 18 from Miray; Miray received 7 meals from Zeynep and 10 meals from Eda. According to the information given, compare the number of meals at the beginning and the number of meals in the last state.

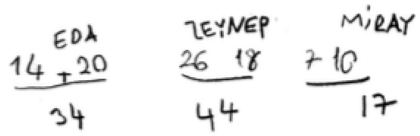

Figure 1. QR1 answer to real life problem with quantitative difference in pre-clinical interview

...

R: Can you explain the meaning of your operations?

QR1: I found the numbers of all

R: What do these numbers mean?

QR1: Meals

R: Can you compare the number of initial meals on Zeynep's card with the number of meals in the last situation?

QR1: 44

...

In the post-clinical interview, she was able to determine the relationships between the quantities required for the problem in the real life problem, which included a quantitative difference, distinguish the quantity unit and interpreted the quantities in a meaningful way. It has been determined that it does not have difficulty in using quantities in accordance with the context of the problem, and handling and manipulating quantities. Below is the student's answer (Fig. 2) and the dialogue section from the clinical interview.

Real life problem with quantitative difference

Erdem, Alper and Neşe are three friends and often meet each other over the internet. If one of them decreases the internet package, the other two friends send additional GB to that person. The amount of GB that three friends get from each other over a period of time is as follows:

(Note: Erdem, Alper and Neşe do not load GB from anywhere else and do not use the internet during this period.)

1) Erdem received 12 from Alper, 18 from Neşe; Alper received 24 from Erdem, 16 from Neşe; Neşe has the right to use 5 GB of internet from Alper and 8 GB from Erdem. According to this information, compare the initial GB amount of Alper and the final GB amount.

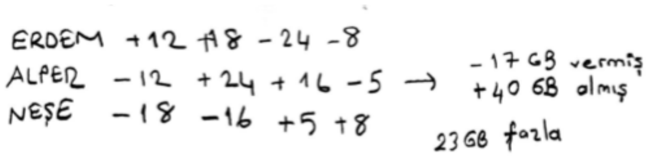

Figure 2. QR1 answer to real life problem with quantitative difference in post-clinical interview

...

R: Can you explain the meaning of your operations?

QR1: I showed GB exchange. If GB is coming to the person, I showed it with a plus sign, if GB is going from that person, it is shown with a minus sign.

R: What do these numbers mean?

QR1: the amount of GB

R: Can you compare Alper's initial GB amount with the final GB amount?

QR1: Alper gave a total of 17 GB to his friends, and 40 GB from his friends, so 23GB more than the beginning

...

3.2. The Student who is Regarded to have Moderate Quantitative Reasoning

The following findings were reached under the theme of determining and interpreting the quantities in the problem context of the student who is to have moderate quantitative reasoning.

It was determined that the student had no difficulty in determining the quantities related to the problem in the pre-clinical interview, he could realize the quantifiable properties and unit of quantities, distinguish the quantities and units of quantities, and interpreted the quantities related to the solution of the problem in a meaningful way (expressing quantities by considering the object, the object's measurable property and its unit together). In the clinical interview after the application, it was found that the same actions can be performed again.

The following findings were reached under the theme of identifying quantitative relationships in the context of the problem and analyzing the change of quantities.

The student had difficulty in making multiplicative and proportional judgments about the relations between the quantities in order to find the unknown value and estimate a series of calculations before performing the calculations in the pre-clinical interview, and it was not difficult to establish the relations between the quantities in the context of the problem. It has been found that a model, formula and algebraic sentence can express and represent all the quantities and the relations between quantities. It has been determined that it can analyze the change of quantities in the context of the problem and interpret the value of one variable together with the change in the other variable. In the post-application clinical interview, it was found that it is no longer difficult to make multiplicative and proportional reasonings about relationships between quantities to find the unknown value and estimate a series of calculations before making calculations. At the same time, it was observed that the actions carried out in the preliminary interview can be carried out again.

The following findings were reached under the theme of identifying and calculating quantitative operations in the context of the problem.

It was determined that the student could not use the quantities in accordance with the problem context in the pre-clinical interview and had difficulty in handling and manipulating the quantities. It has been revealed that he knows how to calculate the quantities in the context of the problem and can describe this situation. It has been determined that it is difficult to determine the meaning of the calculated value and how it is related to the purpose of the problem. In the post-application clinical interview, in terms of problems involving complex situations, quantities are used in accordance with the context of the problem, it is possible to process quantities, how to calculate the quantities in the context of the problem, and how they can describe this situation, It is observed that the calculations can be explained and the meaning of the calculated value and how this value relates to the purpose of the problem can be determined.

The following findings have been reached under the theme of interpreting, controlling and inferring quantitative information with the context of the problem.

In the pre-clinical interview, it was observed that the student could not review and retest every aspect of the solution plan in terms of quantitative relationship. It has been determined that the calculations made in terms of the relationship between quantities and quantitative coordination are not controlled, and when it finds an unexpected value, the situation in the context of the problem is not returned. It has been found that by reference to the quantities and the problem context in the context of the problem, the rationale for the correctness of the solution cannot be presented or an argument cannot be created. In the post-application clinical interview, it is possible to review every aspect of the solution plan in terms of quantitative relationship, including problems involving complex situations, and to re-test, to calculate the relationship between quantitative and quantitative coordination, to return to the situation in the context of the problem when it finds an unexpected value. Also by referring to the quantities and problem context in the context of the problem, it was found that the justification for the correctness of the solution was presented or an argument was created.

3.3. The Student who is Regarded to have Strong Quantitative Reasoning

The following findings were reached under the theme of determining and interpreting the quantities in the context of the problem of the student who is regarded to have strong quantitative reasoning.

It was determined that the student had no difficulty in determining the quantities related to the problem in the pre-clinical interview, she could realize the quantifiable properties and unit of quantities, distinguish the quantities and units of quantities, and interpreted the quantities related to the solution of the problem in a meaningful way (expressing quantities by considering the object, the object's measurable property and its unit together). In the clinical interview after the application, it was found that the same actions can be performed again.

The following findings were reached under the theme of identifying quantitative relationships in the context of the problem and analyzing the change of quantities.

The student was able to make relational, multiplicative and proportional judgment about the relations between quantities, to find the unknown value and estimate a series of calculations before making calculations in the pre-clinical interview, and that it was not difficult to establish relations between the quantities in the context of the problem. It has been found that a model, formula and algebraic sentence can express and represent all the quantities and the relations between quantities. It has been determined that it can analyze the change of quantities in the context of the problem and interpret the value of one variable together with the change in the other variable. In the clinical interview after the application, it was found that the same actions can be performed again.

The following findings were reached under the theme of identifying and calculating quantitative operations in the context of the problem.

It was determined that the student used the quantities in accordance with the problem context in the pre-clinical interview and did not have difficulty in handling and manipulating the quantities. It has been revealed that he knows how to calculate the quantities in the context of the problem and can describe this situation. It was determined that it can determine the meaning of the calculated value and how it is related to the purpose of the problem. In the clinical interview after the application, it was found that the same actions can be performed again.

The following findings have been reached under the theme of interpreting, controlling and inferring quantitative information with the context of the problem.

In the pre-clinical interview, it was observed that the student could not review and retest every aspect of the solution plan in terms of quantitative relationship. It has been determined that the calculations made in terms of the relationship between quantities and quantitative coordination are not controlled, and when it finds an unexpected value, the situation in the context of the problem is not returned. It has been found that by reference to the quantities and the problem context in the context of the problem, the rationale for the correctness of the solution cannot be presented or an argument cannot be created. In the post-application clinical interview, it is possible to review every aspect of the solution plan in terms of quantitative relationship, including problems involving complex situations, and to re-test, to calculate the relationship between quantitative and quantitative coordination, to return to the situation in the context of the problem when it finds an unexpected value. Also by referring to the quantities and problem context in the context of the problem, it was found that the justification for the correctness of the solution was presented or an argument was created.

3.4.Quantitative Reasoning Structures of Group is Examined in the Modeling Process

In the step of understanding the problem, they made very effective discussions in the name of quantitative reasoning in the process where they identified the real life problem and collected and analyzed the data required for the problem. Especially in this process, the important development of the student who has poor quantitative reasoning has been determined under the theme of determining and interpreting the quantities in the context of the problem. An example discussion dialogue supporting this view is given below.

In the modeling activity called meatball wars, there are data that includes the earnings of two meatballs in a neighborhood on the sale of Crab Meatball (Yengeç Köfte) and Lobster

Meatball (Istakoz Köfte) for promotion purposes. In the light of these data, the question is to find the best promotional strategy for each meatball restaurant and explain in detail why these strategies are the best strategies. Group answer to this question is given in Fig. 3.

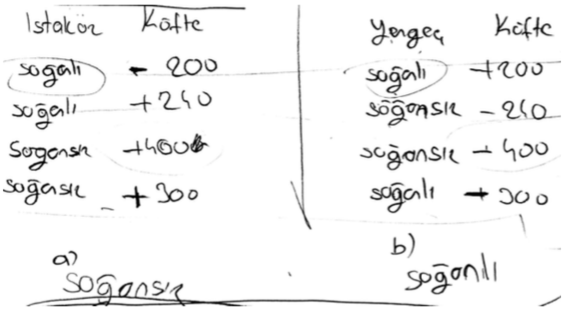

Figure 3. Group answer to the modeling activity called meatball wars

...

QR3: Let's write the profit and loss situations of each meatball shop according to sales with and without onions.

QR1: How do we write

QR3: Let's show with plus sign when they profit, and we show loss with minus sign. For example, Crab Meatball (Yengeç Köfteci), when they sell with onions, is 200 TL profit. Write Crab (Yengeç) and write onion with 200 plus

QR1: got it

...

QR1: Decide the best strategy says how do we decide?

QR3: When does the crab meatballs be profitable?

QR2: Buying with onions

QR1 Exactly then they should definitely sell onions

...

R: What do these numbers mean?

QR1: Profit-loss situations, right?

QR3: Yes

...

In the step of selecting the variables, the important development of the student who has poor quantitative reasoning in the process in which they reviewed certain characteristics of the problem by brainstorming, created a list of features and defined the variables to be used in the model, and analyzed the change of the quantities was determined. It has been observed that the student, who has moderate quantitative reasoning, takes part in discussions that support the process of doing multiplicative and proportional reasoning related to the relations between the quantitative under the same theme. An example discussion dialogue supporting this view is given below.

In the modeling activity called footprint, a thief is tried to be estimated according to the shoe size. The height and shoe sizes of 35 people are given and a general method of finding the height of a person according to shoe size is asked.

...

QR2: Let's subtract shoe sizes from their height, if the difference is close, we can find it accordingly.

QR1: I think it's a good idea

QR3: I think the difference is not meaningful. If we look at the ratio between them, I think we will comment better.

QR2: So

QR3: Let's divide the length by shoe size

QR2: Good idea

...

In the steps of establishing a mathematical model and solving a mathematical problem, in line with the assumptions to be made, it has poor and moderate quantitative reasoning in the process in which they formulate a mathematical model that will represent or define the real life situation by establishing mathematical structures such as graphs, equations, inequalities, and solving the problem. The important developments of the students who are in the context of the problem are determined under the theme of determining the quantitative operations and making calculations. An example discussion dialogue supporting this view is given below.

In the modeling activity, which is the best of gasoline, information is given including the type of gasoline, the amount of gasoline, the price paid on the road traveled. They are asked to mathematically express the relationship between the amount of gasoline received and the money paid. Group answer to this question is given in Fig. 4.

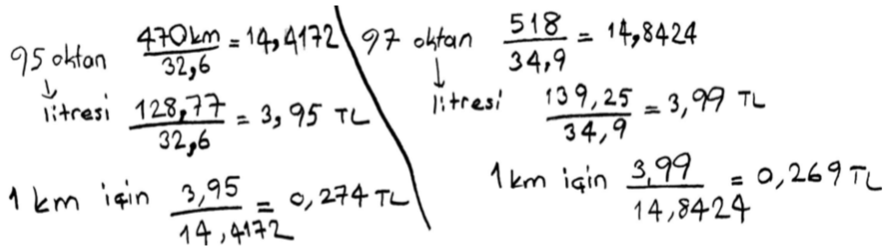

Figure 4. Group answer to the modeling activity called which is the best of gasoline

...

QR1: 470km traveled with 95 octane and a total of 32.6 liters spent

QR2: how much liter of these gases

QR3: Let's find it first

...

QR3: Let's see how many TLs per km

QR2: 3.95 TL per liter

QR3: Approximately 14km of road were traveled

QR2: Let's rate

QR3: Accordingly, we can decide the best of gasoline

...

In the steps of interpreting the solution, validating the model and developing the model for other problems, they evaluate the results of the mathematical analysis, decide the data needed to approve the model, test the ideality of the model using appropriate data, question the results of the model, review the assumptions, and re-formulate the model. During the process of solving, interpreting and approving, important developments of three students were identified under the theme of interpreting, controlling and making conclusions with the context of the problem. An example discussion dialogue supporting this view is given below.

In the modeling activity called footprint, a thief is tried to be estimated according to the shoe size. The height and shoe sizes of 35 people are given and a general method of finding the height of a person according to shoe size is asked. Group answer to this question is given in Fig. 5.

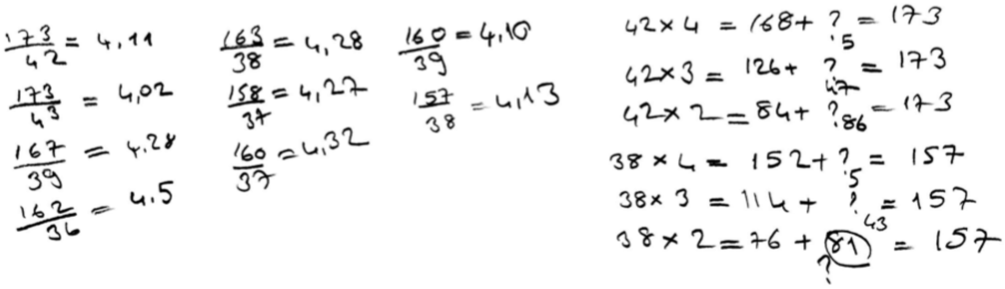

Figure 5. Group answer to the modeling activity called footprint

...

QR1: What are we going to do now?

QR3: So there are many options

QR2: Like 4 times

QR3: If we take 4 times we can add about 5 cm

QR2: Can we get it in 3 times?

QR3: We take it, then we add about 40 or 45 cm

QR2: If we take 2 times, we add about 80 cm

QR1: Which one will we get?

R: You can also add the information of people you know around you to the data

...

QR3: When we think about the people around us, it seems like 2 times more meaningful.

QR1: I think it's more convenient

QR2: Then if we take 2 times

QR3: We can think of the height like 2 times the size of the foot plus 80 cm.

…

4. Discussion, Conclusions and Recommnendations

Quantitative reasoning is a way to describe a student's mental actions about making sense of a mathematical problem situation, creating quantities in this context, and then associating them, manipulating them, and using these quantities in accordance with the problem situation (Weber, Ellis, Kulow & Ozgur, 2014, s. 25). According to Moore, Carlson and Oehrtman (2009), quantitative reasoning includes mental actions such as understanding students' problem context, creating / determining quantities related to this context, establishing relationships between created / determined quantities, analyzing the change between quantities, and it helps to examine by building a structure. Research shows the importance of gaining quantitative reasoning skills especially for students at elementary school level (Ellis, 2007; Smith & Thompson, 2007). Most of the studies with elementary school students in the process of algebra have shown that students whose quantitative reasoning skills are not able to interpret problems that require reasoning, such as daily life problems, tend towards arithmetic instead of establishing inter-quantitative relationships, but students who have quantitative reasoning skills do not even need symbolic representations in algebraic problems (Kabael & Akın, 2016; Smith & Thompson, 2007). Considering that problems based on daily life experiences have an important role in the development of quantitative reasoning, this study aims to examine how the modeling process affects the development of the quantitative reasoning structures of the students.

The findings show the important developments in the reasoning structures of students with poor, moderate and strong quantitative reasoning. Students with poor quantitative reasoning showed very important developments are in the themes of (i) identifying and interpreting quantities in the context of the problem, other than complex situations, (ii) determining quantitative relationships in the context of the problem and analyzing the change of quantities, (iii) determining and calculating quantitative processes in the context of the problem, (iv) interpreting quantitative information with the context of the problem, checking and making conclusions. Students with moderate quantitative reasoning developed significant quantitative reasoning skills in the theme of (i) identifying and calculating quantitative processes in the context of the problem, and (ii) interpreting, controlling and concluding quantitative information in the context of the problem. Students with strong quantitative reasoning, on the other hand, showed significant improvements in the theme of (i) interpreting, controlling and concluding quantitative information in the context of the problem. In the light of the findings, it can be said that the modeling process improved the students' quantitative reasoning structures at all levels. Given the development at all levels, it demonstrates the study's relevance in typical classroom settings. Furthermore, the modeling process is a process that supports quantitative reasoning in the sense that students think aloud while solving problems, explain all of their approaches and thoughts, establish a connection between the situations in real-life problems, reveal a dynamic relationship, and finally make the desired relationships in real life problems.

Quantitative reasoning, on the other hand, is not given the attention it deserves in the classroom. While the emphasis in elementary school mathematics is on numbers, arithmetic, and operations, quantitative reasoning is undervalued. In addition, curricula are insufficient for students to develop a connection between daily life and mathematics (Smith & Thompson, 2007). The algebra learning area in the elementary mathematics curriculum allows students to not only understand, manipulate, and use the algebraic notational system, but also to shape methods of thinking about quantitative and numerical relationships (Thompson, 1990). In Turkey, a direct emphasis on quantitative reasoning is placed in the curriculum, just thinking of the students in the problem-solving process and their ability to comfortably express their reasoning. It is also emphasized that students should be able to see the flaws or gaps in others' mathematical reasoning (MoNE, 2018). According to research, textbooks and curricula do not promote quantitative reasoning (Smith & Thompson, 2007; Stigler, Fuson, Ham & Kim, 1986). Teaching students mathematics in a strong and productive way depends on giving importance to the development of quantitative reasoning. Therefore, students' reasonings about quantitative situations and actions related to conceptualization of their skills should not be less important than computational skills (Smith & Thompson, 2007). In mathematics teaching, no approach will assist students acquire concepts and abilities unless quantitative reasoning is prioritized (Thompson, 1990). Many teachers find teaching quantitative reasoning to be challenging, and the vast majority of teachers are unable to apply quantitative reasoning successfully (Post, Harel, Behr & Lesh, 1991). From this point of view, mathematics teachers and prospective mathematics teachers need to teach only in the solution of the problem, in which they do not focus only on procedural procedures, but ignore the mental needs and ways of thinking of students. However, it should not be forgotten that solving problem situations rich in quantitative relations does not mean that students will improve their problem solving skills and ways of thinking. The reason is that students often focus on transactions and numbers in case of problems. Students may only produce and link quantities in problem situations if the mathematics teacher provides appropriate and well-structured directions and queries to support their quantitative reasoning. As a result, teachers working with students' reasoning structures and striving to establish an understanding based on the links between quantities and quantities may better prepare their students to solve algebraic verbal problems during the problem-solving process. From this perspective, the fact that mathematical modeling is a required course in Turkey's primary school mathematics curriculum and that quantitative reasoning is addressed in this course is a significant step forward. The study's limitation is the number of students who participated in it. In future studies, it is recommended to investigate quantitative reasoning structures with students at different levels and with different number of participants in the modeling process.

5. Conflict of Interest

The author declares that there is no conflict of interest.

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