International Online Journal of Education and Teaching (IOJET), Vol. 11 , No. 1, 2024 (pp. 125-137)

RESEARCH ARTICLE https://IoJetJournal.com

Evaluation of Geometry Textbooks Through the Cognitive Point of View

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Keywords:
Geometry education, Assessment and evaluation, Cognitive domains, TIMSS 2019 assessment framework

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Received : 08.07.2023

Revised : 04.11.2023

Accepted : 06.11.2023

Abstract

The importance of cognitive approach in geometry education is increasing. In this direction, curriculum programs are renewed. One of the guides that teachers consider when applying the renewed curriculum in their classrooms is textbooks. Textbooks are used in many areas such as the order of the subjects, their boundaries and assessment and evaluation questions. From this point of view, the aim of the study is to evaluate geometry textbooks from a cognitive perspective. The research was carried out as a qualitative descriptive research. The sample of the research consists of two different 9th grade mathematics textbooks accepted after the last update. The assessment and evaluation questions in the textbooks and the tools, equipment, DGS and daily life examples used in the contents were evaluated separately by document analysis. Assessment and evaluation questions TIMSS-2019 assessment framework was used. As a result, it was determined that almost all of the questions used were in the application cognitive domain. It was observed that the obtained values were different from the ratios determined by TIMSS. It was concluded that a limited number of tools and equipment were used in the contents.

How to cite this article: İnam B, Turanlı N (2024). Evaluation of Geometry Textbooks Through the Cognitive Point of View. International Online Journal of Education and Teaching, Vol. 11, No. 1, 2024, 125-137

Introduction

Geometry is included in mathematics teaching programs at all grade levels, from primary school to university, using varying subjects and different teaching approaches. Due to this wide scope, it is inevitable to conduct researches with different focuses in the field of geometry education. Torregrosa and Quesada (2008) stated that in recent years, research on geometry teaching and learning processes concentrates on two main areas: (1) proofs and different production ways of proofs and (2) the development of cognitive processes that occur when students solve geometric problems. This study is based on the idea that being aware of the cognitive processes that take place in the acquisition of geometric knowledge is especially important for educators who teach geometry.

Different approaches have been proposed to explain the cognitive structure of geometry. Schoenfeld (1986, p.225) described geometric objects as shapes suitable for performing both physical

operations and formal mathematical operations in what he called "cognitive support structure". Due to this structure, he stated that deductive and experimental approaches should be used together in the process of understanding geometric knowledge.

Fischbein (1993, p.139) explained the cognitive structure of geometric knowledge with the "figural comprehension" model. According to Fischbein, geometry is a field that deals with mental geometric shapes that have both conceptual and figural character at the same time. Geometric knowledge consists of the combination of the geometric concept formed in the human mind and the geometric figure drawn as the representative of that concept. He stated that the relationship between the concept formed in the mind and the shape will determine the geometric reasoning processes that may arise.

Another study based on the cognitive approach belongs to Duval. Duval proposed a model in which he described two dimensions as perceptual and cognitive and the sub-processes of these dimensions. In the perceptual dimension (Duval, 1995), he stated that there are four different perception processes that allow a geometric shape to be noticed and interpreted and to perform operations with this shape. The cognitive dimension includes three types of cognitive processes called visualization, construction and reasoning (Duval, 1998). Duval argues that learning mathematics will occur through the construction of an object's cognitive structure; he claims that when information is presented directly, learning is either impossible or seldom possible (Duval, 2007, p.153).

The cognitive dimension in geometry education is also included in the publications of the institutions that are taken into account in the preparation of the teaching content. The National Council of Teachers of Mathematics (NCTM) stated that judgment, construction and communication skills should be at the center of the study of geometry (NCTM, 2000). Similarly, the frameworks used in the International Trends in Mathematics and Science Studies (TIMSS) exam are organized around two dimensions: the content dimension and the cognitive dimension. The cognitive dimension is divided into three sub-dimensions: knowing, applying and reasoning. Detailed information about these sub-dimensions will be given in the next section.

The results obtained from the educational researches described also affect the renewal studies in the curriculum. It is aimed to be in a structure that directs the use of metacognitive skills in the mathematics curriculum being implemented in Turkey (MoNE, 2018, p.4). While curricula are being implemented in classrooms, it has been stated that students' mathematical knowledge structuring processes should be supported with multiple representations and materials (MoNE, 2018).

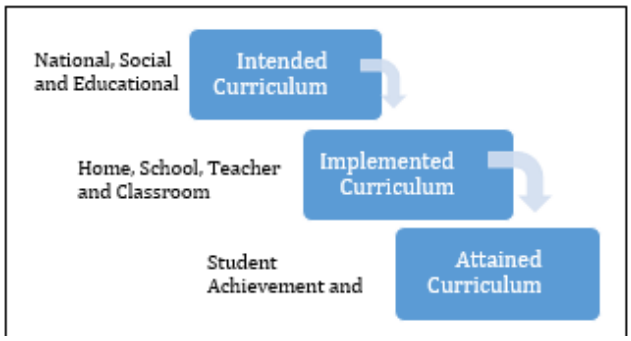


Fig.1: TIMSS Curriculum Model (Mullis & Martin, 2017

In addition to the general and specific objectives expected to be achieved in mathematics education, the subjects to be covered in the lessons and the subject boundaries are also determined in education programs. The curriculum in Turkey is prepared by the Ministry of National Education (MoNE) and the same program is applied in all schools across the country. However, it is a possible result that the degree of achieving the goals set in the curricula differs depending on many reasons such as school type and student level. At this point, TIMSS has introduced a model in which curricula are handled from three perspectives; *"intended curriculum, implemented curriculum, attained curriculum"* (Figure-1). Intended curriculum; the curriculum determined by the ministry, the implemented curriculum; the programs applied in the classrooms, and the attained curriculum refers to the success and attitude levels achieved by the students at the end of the teaching activities. Evaluation of each type of curriculum is made using different criteria. The focus of this study is the implemented curriculum.

Table 1: Functions of Textbooks

Functions	Literature examples
Curriculum Transfer Tool	*"Transfer of an ideal curriculum"* (Usiskin 2013).
	"A way to explore curriculum reform" (Ball & Cohen, 1996).
	"A tool for curriculum change" (Usiskin 2013).
	"Curriculum development was carried out through textbooks" (Howson, 2013).
Determinant of Content to Teach	*"Explanation of math content"* (Usiskin 2013)
	"Textbooks are also a way for educators to strive for a common curriculum in a variety of settings." (Ball & Cohen, 1996)
	"Provides the skeleton of a course"(Howson,2013)
	"Description of the mathematics to be taught" (Howson,2013)
	"It provided teachers with a coherent framework to guide their work."(Howson, 2013)
	"Resource for teachers" (Reys, Reys, & Chavez, 2004)
	"Textbooks provide a scope and sequence in which teaching can be planned" (Usiskin,2013)
Resource for Students	*"It gives the student a picture of what they are expected to learn."* (Usiskin, 2013)
	"Tools with which students interact with math" (Haggarty & Pepin, 2002)
	"Textbook explanations can help students or those trying to help them better understand what the teacher is telling in class." (Howson,2013)
Assessment and evaluation	*"Presentation of problems (with or without solution)"* (Usiskin 2013)

The delivery of the implemented curriculum to the teachers is done through different means such as official notification, in-service courses, and written documents. Textbooks are also written materials in printed form designed to assist in the implementation of the implemented curriculum. The textbook and the implemented curriculum are often seen as so similar that they are used as a determinant of the subjects to be taught in the classroom (Usiskin, 2013). It is thought that many features of textbooks are effective in the emergence of this situation. Many studies have been carried out on textbooks from different perspectives. A literature synthesis on the tasks of textbooks is presented in Table-1.

Related Studies

First of all, research on the general structure of textbooks was examined. Walbesser (1973) stated that the first textbooks in the modern sense were published in America since 1729. He stated that the structure of these textbooks is "giving the rule, presenting examples where the rule is applied, and finally proposing a formal proof". The author stated that the "rule-examples-implementation technique" still continues as of 1973, when the study was published. Similarly, Howson (2013) stated that since the 1990s in England, "kernels", that is, definitions, rules, theorems or features that students need to remember or use, have been given "in the box" or using bold fonts.

There are also researches in which the reflections of the changes made in the curriculum in the textbooks are examined from a cognitive perspective. One of these studies was conducted in order to compare the cognitive domains of 4th grade (10 years old) mathematics textbooks developed within the scope of the old and new curriculum in Malaysia (Tan, İsmail, & Abidin, 2017). In comparison, the analysis of the questions in the textbooks was carried out based on the TIMSS-2015 framework. At the end of the research, they determined that although improvements were made in the "applying" and "reasoning" levels in the new book, most of the mathematical tasks in the two textbooks were in the field of "knowing". Khodaria, Maharani, and Sulaiman (2019) analyzed the questions in two different 11th grade mathematics textbooks

developed as a complement to the curriculum reform that took place in Indonesia in 2013, using the TIMSS 2015 framework. The questions in one of the examined books; they found that 3.5% knowing, 4.71% applying, 91.76% reasoning level, 10.53% applying and 89.47% reasoning level in the other. They stated that these rates are not in accordance with the rates specified by TIMSS. In another study, Rasyidi and Winarso (2020) examined the cognitive aspects of 10th grade mathematics textbooks based on Marzano taxonomy. They found that 17% of the questions in the books they examined were at the level of remembering, 14% at the level of understanding, 30% at the level of analysis, and 39% at the level of cognition. According to the results of the research, they stated that the cognitive distribution in the textbooks they examined was not ideal compared to the proportional distribution of cognitive aspects.

Theoretical Framework

The TIMSS-2019 assessment framework was used to examine the geometry questions in the 9th grade textbooks from a cognitive perspective (Table-2). According to this framework, cognitive fields in mathematics; it has been analyzed by dividing it into three areas as knowing, applying and reasoning. Knowing; *"covers the facts, concepts and procedures that students need to know."* Applying; *"focuses on students' ability to apply knowledge and conceptual understanding to solve problems or answer questions."* Reasoning; *"goes beyond the solution of routine problems to include unusual situations, complex contexts and multi-step problems"* (Mullis & Martin, 2017, p.22). Within the assessment framework, the expected behaviors belonging to each cognitive domain were classified. The cognitive domains included in the framework are also valid for different grade levels; it was stated that the difference was only in the emphasis on cognitive areas (Mullis & Martin, 2017, p.14). For example, it was stated that while less emphasis was placed on *"knowing"* in 8th grades than in 4th grades, more emphasis was placed on *"reasoning"*.

Statement of the Problem

The cognitive perspective in geometry education comes to the fore both in academic publications and in international exams. In this direction, renewal

Table 2: TIMSS-2019 Assessment Framework (Mullis & Martin, 2017)

Cognitive Domains	Cognitive Skill	Expected Behavior
KNOWING	*Recall*	*"Recall definitions, terminology, number properties, units of measurement, geometric properties, and notation (e.g., a × b = ab, a + a + a = 3a)."*
	Recognize	*"Recognize numbers, expressions, quantities, and shapes. Recognize entities that are mathematically equivalent"*
	Classify/Order	*"Classify numbers, expressions, quantities, and shapes by common properties."*
	Compute	*"Carry out algorithmic procedures for +, -, ×, ÷, or a combination of these with whole numbers, fractions, decimals, and integers. Carry out straightforward algebraic procedures."*
	Retrieve	*"Retrieve information from graphs, tables, texts, or other sources."*
	Measure	*"Use measuring instruments; and choose appropriate units of measurement."*
APPLYING	*Determine*	*"Determine efficient/appropriate operations, strategies, and tools for solving problems for which there are commonly used methods of solution."*
	Represent/ Model	*"Display data in tables or graphs; create equations, inequalities, geometric figures, or diagrams that model problem situations; and generate equivalent representations for a given mathematical entity or relationship"*
	Implement	*"Implement strategies and operations to solve problems involving familiar mathematical concepts and procedures."*

Cognitive Domains	Cognitive Skill	Expected Behavior
REASONINGC	*Analyze*	*"Determine, describe, or use relationships among numbers, expressions, quantities, and shapes."*
	Integrate/ Synthesize	*"Link different elements of knowledge, related representations, and procedures to solve problems"*
	Evaluate	*"Evaluate alternative problem solving strategies and solutions"*
	Draw Conclusions	*"Make valid inferences on the basis of information and evidence."*
	Generalize	*"Make statements that represent relationships in more general and more widely applicable terms"*
	Justify	*"Provide mathematical arguments to support a strategy or solution"*

studies are carried out in curricula. New textbooks were written within the scope of these studies. However, studies on the suitability of these textbooks for cognitive domains have not been conducted. On the other hand, no studies were found in which an assessment tool was used to evaluate the questions in the books in cognitive domains.

Purpose of the Study

It is inevitable for textbooks to be a reference for both students and teachers in line with the listed tasks and intended use. However, studies examining textbooks from a cognitive perspective have not been conducted yet. The aim of the study is to evaluate the textbooks, which are representative of the intended curriculum, from a cognitive perspective. For this purpose, assessment and evaluation questions and the materials and tools used in the books were analyzed with the relevant frameworks.

In this study, 9th grade geometry textbooks were examined from a cognitive perspective. Answers to the following research problems were sought.

1) Which cognitive behaviors are expected from the students in the assessment and evaluation questions in the textbook?
2) To what extent is the distribution of the questions in the books in the cognitive domains compatible with the TIMSS-2019 assessment criteria?
3) Which tools and real-life situations are included in geometry textbooks?

Methodology

This research was conducted as a qualitative descriptive study examining the assessment and evaluation questions in 9th grade mathematics textbooks (Lambert & Lambert, 2012). The distribution of the questions in the textbooks according to the cognitive domains was made with the TIMSS-2019 assessment framework. In order to determine the tools, materials and dynamic geometry software used in the questions in the books, the multiple representation framework developed by the researchers was used.

Participants

The focus of the research is two mathematics textbooks, which were published by the Ministry of National Education after the change in the mathematics curriculum in 2017 and distributed to 9th grades all over the country (Table-3).

In the next step, the code K1 will be used for the MoNE publications book, and K2 code will be used for the book belonging to Esen Publishing.

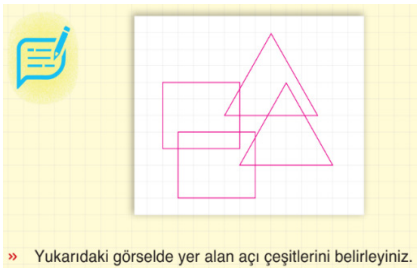
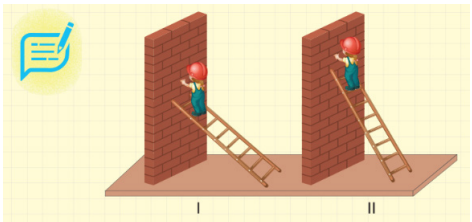
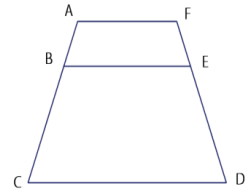
Analysis of Data

TIMSS-2019 assessment framework was used in the analysis of assessment and evaluation questions. At the beginning of the data analysis process, an analysis chart developed by the researchers and based on the TIMSS-2019 framework was created. In the first stage, the researchers completed the analysis independently by using the chart, and then the results were compared and the final version was given.

Table 3: Textbooks constituting the sample of the research

Acceptance Date	Writers	Publisher	Topics
2018	Mehmet MAVİŞ, Güray GÜL, Himmet SOLAKLIOĞLU, Hakan TARKU, Fatih BULUT, Mahmut GÖKŞEN	MONE	☛ Basic Concepts in Triangles, ☛ Concurrency and Similarity in Triangles, ☛ Auxiliary Elements of Triangle, ☛ Right Triangle and Trigonometry, ☛ Area of Triangle
2018	Havva AYTAR	ESEN	

Table 4: Chart and Analysis Examples Used in Analysis of Questions

Tag	Question	Cognitive Domains	Cognitive Skill	Expected Behavior
Book K2 Page p-212	 » Yukarıdaki görselde yer alan açı çeşitlerini belirleyiniz.	Knowing	Classify/ Order	Classification of shapes according to their common feature
Book K2 Page p-223	 Boyacı, duvarın üst kısımlarına ulaşmak için I. konumdaki merdiveni II. konuma getiriyor. Merdiven II. konuma getirildiğinde, merdivenin duvarla arasında oluşan açının ölçüsünün I. konumdakine göre küçük olduğu; merdivenin zeminle arasında oluşan açının ölçüsünün de I. konumdakine göre büyük olduğu görülür. » I. ve II. konumda merdivenin duvarla ve zeminle arasında oluşan açılardaki değişim; merdivenin duvara değdiği noktanın yüksekliğini ve merdivenin duvara uzaklığını nasıl etkiler?	Reasoning	Analyze	Identify, explain, or use relationships between shapes and measures
Book K1 Page p-244	 Şekilde A, B ve C; F, E ve D noktaları doğrusaldır. [AF] // [BE] // [CD] dir. |BC| = 3·|AB| |AF| = 6 birim ve |CD| = 22 birim ise |BE| nu bulunuz.	Applying	Implement	Application of geometric strategies and operations to solve the problem.

The developed chart and analysis examples are presented in Table-4.

The framework developed by the researchers was used to determine the materials, materials and DGS

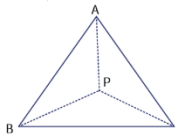
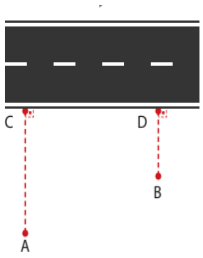
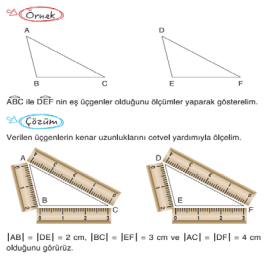
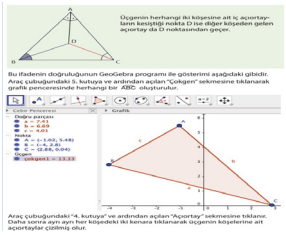
Tag	Question	Cognitive Domains	Cognitive Skill	Expected Behavior
Book K1 Page p-274	 Şekildeki P noktası $\widehat{ABC}$ nin kenar orta dikmelerinin kesiştiği noktadır. $\|PA\|$, $\|PB\|$, $\|PC\|$ arasındaki ilişkiyi gösteriniz.	Reasoning	Draw Conclusions	Make valid inferences using knowledge and theorems.
Book K1 Page s-292	 A ve B belediye merkezlerinin düz bir otoyola dik uzaklıkları sırasıyla 10 km ve 6 km dir. Otoyol bağlantılı olacak şekilde A ve B belediye merkezlerini birbirine bağlayan asfalt yol yapılacaktır. $\|CD\|$=12 km ve bu yol için dökülecek asfalt yolun 1 km lik uzunluğunun maliyeti 20 000 Türk lirası ise **en az** kaç Türk lirası masraf edileceğini bulunuz.	Applying	Represent/ Model	Creating a geometric figure that models the problem situation

Table 5: Analysis Framework of Multiple Representations Used in Content

Category of Multiple Representation	Content
Tools	It includes geometric drawing tools such as compass, ruler, grapier (non-measured ruler), and miter.
Dynamic Geometry Software	It covers all software that can be run on computers, tablets or phones and has drawing features, such as GeoGebra, Cabri.
Materials	It includes materials used other than formal mathematical symbols such as tangram, cardboard, scissors, geometry tiles, a picture from daily life.

Table 6: Charts and Analysis Examples Used in the Analysis of Representations

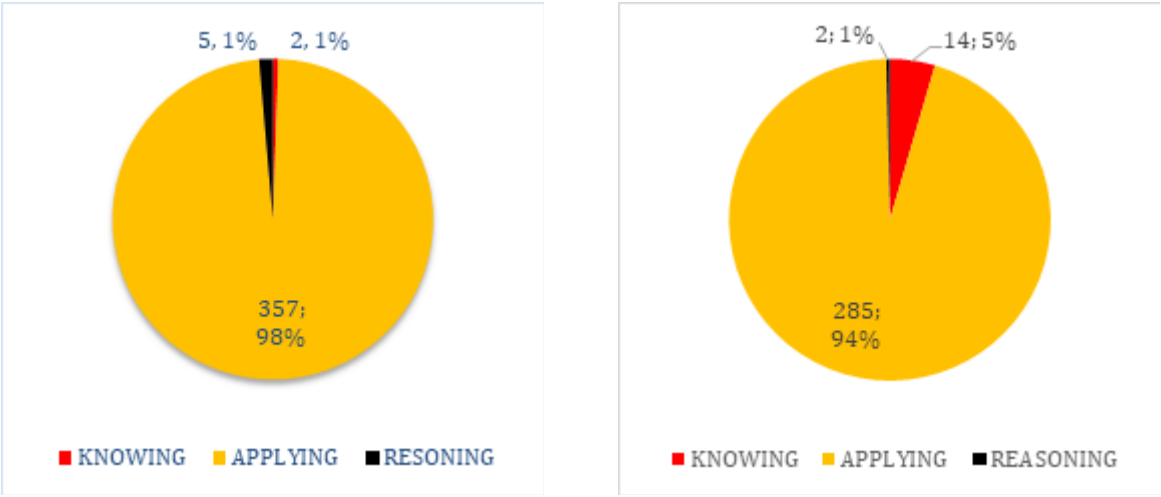
Tag	Example	Category	Content
Book K2 Page 238		Tools	Using ruler
Book K1 Page 254		Dynamic Geometry Software	GeoGebra

used in the contents and questions in the textbooks examined (Table-5).

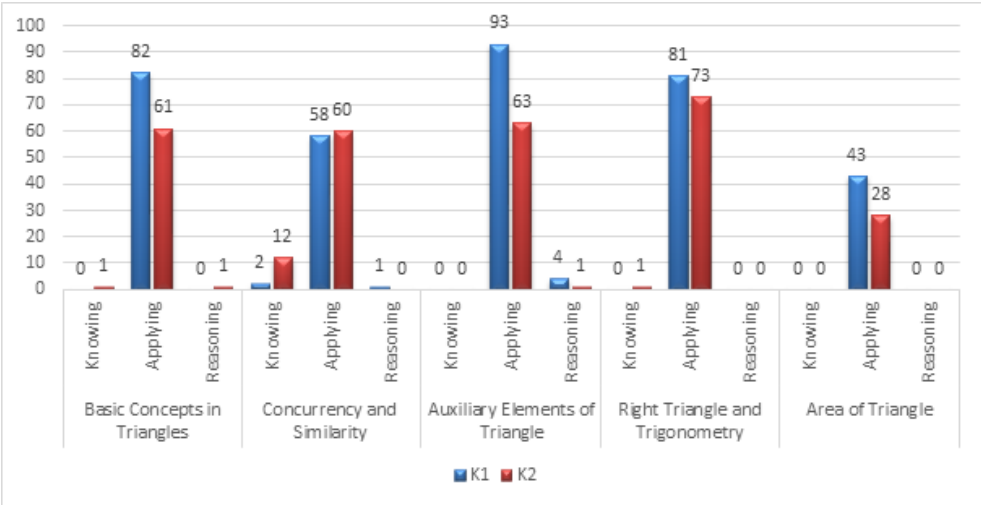
The analysis chart and analysis samples are presented in Table-6.

Findings

The results of the analysis of the cognitive domains of the questions in the textbooks are presented in Graph 1.



Graph 1: Distribution of questions in the examined books according to cognitive domains



Graph 2: Distribution of the questions according to the cognitive domains on the basis of subject

Although the number of questions included in the books is different, it has been determined that the distribution of the questions according to the cognitive areas is close. The results of the comparative analysis of the questions in the textbooks for the cognitive domains on the basis of subject are presented in Graph-2.

It was determined that there were differences in the distribution of questions on the basis of units in the books. For example, while there are 97 questions in the book K1 about the auxiliary elements of the triangle, there are 64 questions in the book K2. On the other hand, there are more questions in the K2 book about triangle congruence and similarity.

Another finding is that the questions about all cognitive domains were not included in the subjects. For example, in the book K2, questions from all three cognitive domains about the basic concepts of triangle were included, while questions about the area of the triangle were included only in the cognitive domain of application. A similar situation can be seen in the book K1.

The questions in the books were also examined according to cognitive skills. The obtained results are presented in Table-7.

It was determined that there were no questions about; "compute" and "retrieval" in the cognitive domain of knowing, "determination" in application cognitive domain, "evaluate", "generalize" and "justify" behaviors in the cognitive domain of reasoning.

The analysis results of which tools and real-life situations are used in the lectures and questions of the examined textbooks are presented in Table-8.

The comparison of the course contents and the total number of questions in the examined textbooks with the number of dynamic geometry software, materials or tools used is presented in Table-9.

Results, Discussion and Recommendations

Changes are being made in mathematics curriculum programs in line with both 21st century skills and the criteria announced in international exams. One of the tools that enable these changes to be transferred to teachers and students is textbooks. Textbooks are one of the references used for teachers and students to determine the boundaries and sequence of the subjects to be covered in the lessons. In line with the recent changes made in the curriculum, it is seen that the cognitive dimension is given more place in geometry education. Determining how changes and expectations are reflected in teaching environments

Table 7: Distribution of Questions According to Expected Behavior

Cognitive Domain	Expected Behavior	K1	K2
KNOWING	Recall	-	4
	Recognize	2	3
	Classify/Order	-	1
	Measure	-	6
APPLYING	Represent/Model	27	32
	Implement	330	253
REASONING	Analyze	2	-
	Synthesize	1	2
	Draw Conclusions	2	-

Table 8: Number of tools, materials and DGS used in textbooks

	DGY	MATERIAL		TOOL		
	GeoGebra	Real life Situations	Tangram	Ruler	Protractor	Compass
K1	12	21	-	6	5	2
K2	10	22	1	5	4	2

Table 9: Comparison of content and number of questions with the use of tools, materials and DGS

	Total content and number of questions	Total number of tools, materials or DGS use	Percentage (%)
K1	437	46	10,5
K2	331	44	13,3

is one of the research topics. Textbooks also provide important data at this point.

In the literature, it has been observed that there are no studies in our country in which mathematics textbooks are examined from a cognitive perspective. The TIMSS-2019 assessment framework used in the research is the first in this respect. Within the TIMSS-2019 assessment framework, where the analyzes were made, it was stated that the distribution of the questions according to the cognitive domains should be in the form of knowing (35%), applying (40%) and reasoning (25%) for the 8th grade level. It was stated that as the grade levels progressed, the cognitive areas remained the same and the change should only be in the distribution ratios. Tan, İsmail, and Abidin (2017) stated that in the mathematics textbooks they examined, questions were included in the cognitive domain of knowing, while Khodaria, Maharani, and Sulaiman (2019) stated that questions were included in the cognitive domain of reasoning. Both studies are aimed at examining the textbooks published just after the curriculum change in the countries. It was determined that the assessment and evaluation questions in the textbooks examined within the scope of the research consisted of questions at the application cognitive level to a large extent. In line with this result, it was understood that the assessment and evaluation questions in the examined textbooks were far from ideal ratios in the distribution of cognitive domains.

When the number of questions in the books were compared on the basis of topics, it was determined that there were differences in both the number of questions and their distribution according to cognitive areas. It has been observed that this situation is also valid for different subjects in the same book. It was determined that while questions related to all three cognitive domains were included under one subject, only questions were included in the applying cognitive domain under another subject. This situation may cause differences in terms of education and training unity in schools that use different textbooks. In order to eliminate such differences, it is thought that standards should be developed by the institutions responsible for the preparation of the books.

When the distribution of the questions according to the expected behaviors was examined, it was determined that the majority of them were related to the implement behavior. Although geometry is related to many aspects of daily life, it has been observed that representation and modeling behavior are rarely included in the questions. It is thought that targeting limited behaviors will have a limited effect on the development of students' geometric cognitive skills. Duval (1998) states that in order for students to develop their reasoning skills, it is necessary to enable them to discover how deductive reasoning is organized. In order to achieve this, he states that informing the students about the reasoning process can be helpful. Contrary to this situation, when questions are presented in the form of ready-made rules without explaining cognitive processes, it may be difficult for students to realize their reasoning processes. Similarly, it is seen that content based on the structuring of knowledge should be developed for the development of metacognitive skills, which are included in the MoNE (2018) curriculum and expected to be acquired by students. It has been determined that the questions in the textbooks discussed within the scope of the research are far from these expectations. It was determined that the definitions, rules, theorems or properties that students should remember or use in their solutions were given in the text or on the margins of the page, in a box or using bold font. Similar results were obtained from the study conducted by Howson (2013) on textbooks published in England. The understanding of the "rule-examples-application" technique stated by Walbesser (1973) was also widely seen in the textbooks examined.

It was determined that reasoning methods other than deductive proof were given very little place in the textbooks examined within the scope of the research. It has been observed that a limited number of explanations made with DGY are included, apart from formal proofs. As a result of these distributions, it is not expected that students develop skills such as perceiving the relationships between geometric structures and developing different perspectives. Similarly, Günhan and Açan (2016) in their meta-analysis study in which they investigated the effect of DGS use on geometry success in Turkey, and concluded that the use of dynamic geometry software has a strong effect compared to traditional teaching. In their

study in which they examined mathematics textbooks in terms of technological suitability, Sevimli and Kul (2015) determined that the content that includes the use of technology is few in number, and that these contents are mostly based on the use of calculators to support the calculation skill. In their study, they concluded that the content suitable for the use of technology in mathematics textbooks is mostly aimed at achieving low cognitive goals. Ubuz and Sarpkaya (2014) also stated that the content and methods to support cognitive association in the activities in the 6th grade textbooks are insufficient. On the contrary, the mathematics achievements in TIMMS and PISA exams of countries such as Singapore and South Korea, which update their education policies according to the developments in information technologies, draw attention (Koay, 2006). It has been stated that high-level cognitive skills and processes such as model building, discovery, association, interpretation and analysis should be supported in the integration of information technologies into teaching activities (Engström, 2004).

Another issue investigated in the textbooks is the extent to which daily life situations are included. Realistic mathematics education (RME), whose foundations were laid by Hans Freudenthal, aims to provide students' mathematical development by using real life problems. Geometry is a field that can be associated with many fields such as architecture, art, design and design. It has been demonstrated in many studies that realistic life situations have a positive effect on mathematics achievement (Korkmaz & Tutak, 2017). Özdemir (2020), in his meta-analysis study based on studies examining the effect of RME understanding on mathematics achievement, concluded that RME has a positive and wide-ranging effect on mathematics achievement. Based on the results of his research, the author recommends that MONE include RME-based activities in textbooks. In addition to all these, the inclusion of daily life situations in the activities is a situation revealed in the MoNE (2018) curriculum. Demirdöğen (2007) states that the RME approach can meet the expectations in the program. It was determined that the proficiency of the examined textbooks on this subject was limited and they were only used in right triangle and trigonometry topics.

The recommendations made in line with all these results are listed below:

1. While preparing assessment and evaluation questions, attention should be paid to a distribution suitable for different cognitive areas, age group and course content. Questions about different cognitive domains can provide students with the opportunity to apply various strategies and switch between cognitive processes and cognitive domains. It can also encourage teachers to use alternative assessment tools.
2. The construction processes of geometric shapes using compass, graphite or DGY should be included in the course content. These tools should be included not only to "draw" but to determine "how to draw". Thus, the construction processes of geometric structures can be created in a meaningful way.
3. Examples of daily life should be included in the course content. This situation is not limited to only perpendicular and trigonometry subjects; using geometric shapes in different areas such as designing a new object, constructing a shape in different ways can be beneficial for student development.

Suggestions for future research are listed below:

1. This study is limited to 9th grade textbooks. Research can be done on textbooks at different grade levels.
2. This study is limited to the geometry learning area. Studies can be conducted in which different learning areas are evaluated from a cognitive perspective.
3. The focus of the study is textbooks. For example, cognitive assessment studies can be conducted for different samples, such as the central exams held in our country.

It is thought that all these results and suggestions will increase the quality of the textbooks, thus increasing the competitiveness of the textbooks both nationally and internationally.

Acknowledgement

This article has been generated from the Ph.D. thesis of the first author which was supervised by the second

author at Hacettepe University Graduate School of Educational Sciences in Ankara, Turkey.

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